

**Section I****10 marks****Attempt Questions 1-10****Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for Questions 1-10.

1. Which of the following is equal to  $\frac{x^2 - 36}{x - 6}$  ?

- (A)  $x - 6$
- (B)  $x + 6$
- (C)  $x - 3$
- (D)  $x + 3$ .

2. What are the solutions to  $3x^2 - 7x - 1 = 0$  ?

- (A)  $x = \frac{-7 \pm \sqrt{37}}{6}$
- (B)  $x = \frac{-7 \pm \sqrt{61}}{6}$
- (C)  $x = \frac{7 \pm \sqrt{37}}{6}$
- (D)  $x = \frac{7 \pm \sqrt{61}}{6}$ .

3. What are the exact solutions of  $2 \cos x = -\sqrt{3}$  for  $0 \leq x \leq 2\pi$  ?

- (A)  $\frac{\pi}{6}$  and  $\frac{11\pi}{6}$
- (B)  $\frac{5\pi}{6}$  and  $\frac{7\pi}{6}$
- (C)  $\frac{\pi}{3}$  and  $\frac{5\pi}{3}$
- (D)  $\frac{2\pi}{3}$  and  $\frac{4\pi}{3}$ .

4. Which of the following define the domain and range of the function  $f(x) = \log_e x$  ?

- (A) Domain: all real  $x$  and Range: all real  $y$ .
- (B) Domain:  $x > 0$  and Range:  $y > 0$
- (C) Domain: all real  $x$  and Range:  $y > 0$
- (D) Domain:  $x > 0$  and Range: all real  $y$ .

5. What is the derivative of  $(e^{3x} + 1)^{-2}$  ?

- (A)  $-2e^{3x}(e^{3x} + 1)^{-3}$
- (B)  $-2e^{3x}(e^{3x} + 1)^{-1}$
- (C)  $-6e^{3x}(e^{3x} + 1)^{-3}$
- (D)  $-6e^{3x}(e^{3x} + 1)^{-1}$ .

6. What is the perpendicular distance of the point (4,5) from the line  $3x - 2y + 10 = 0$  ?

- (A)  $\frac{12}{\sqrt{13}}$
- (B)  $\frac{17}{\sqrt{13}}$
- (C)  $\frac{2}{\sqrt{5}}$
- (D)  $\frac{12}{\sqrt{41}}$ .

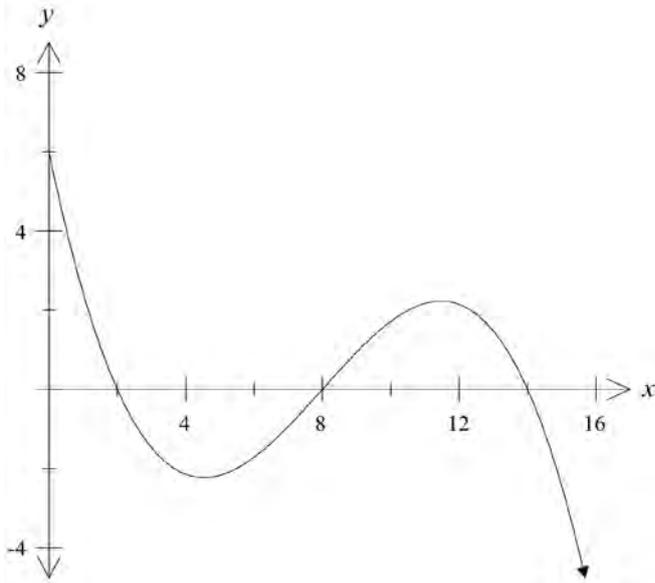
7. What is the solution of  $5^x = 20$  ?

- (A)  $\log_4 5$
- (B)  $\log_5 4$
- (C)  $1 + \log_4 5$
- (D)  $1 + \log_5 4$ .

8. A parabola has a focus  $(3,1)$  and directrix  $x = 5$ . What is the equation of the parabola?

- (A)  $(y-1)^2 = -4(x-4)$
- (B)  $(y-1)^2 = 8(x-3)$
- (C)  $(x-3)^2 = -8(y-3)$
- (D)  $(x-3)^2 = -16(y-1)$ .

9. The diagram below shows the graph  $y = f(x)$ .



Where is the function increasing, at a decreasing rate?

- (A)  $(2,0)$
- (B)  $(6,-1.8)$
- (C)  $(10,1.8)$
- (D)  $(14,0)$ .

10. What is the value of  $\int_3^5 4x^3 dx$

- (A) 192
- (B) 408
- (C) 544
- (D) 706.

**Section II****90 marks****Attempt Questions 11-16****Allow about 2 hours and 45 minutes for this section**

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Question 11-16, your responses should include relevant mathematical reasoning and /or calculations.

**Question 11 (15 marks)**

- a) Evaluate  $\frac{\ln 5}{3}$  correct to three significant figures. **1**
- b) Evaluate  $\lim_{x \rightarrow 3} \frac{x^3 - 3x^2}{x - 3}$  **2**
- c) Differentiate  $(1 + \tan x)^4$  **2**
- d) Differentiate  $x \ln x$  **2**
- e) Find  $\int 4xe^{x^2+1} dx$  **2**
- f) Evaluate  $\int_0^2 \frac{3x}{x^2+1} dx$  **3**
- g) Sketch the region defined by  $x^2 + (y-1)^2 \geq 9$  **3**

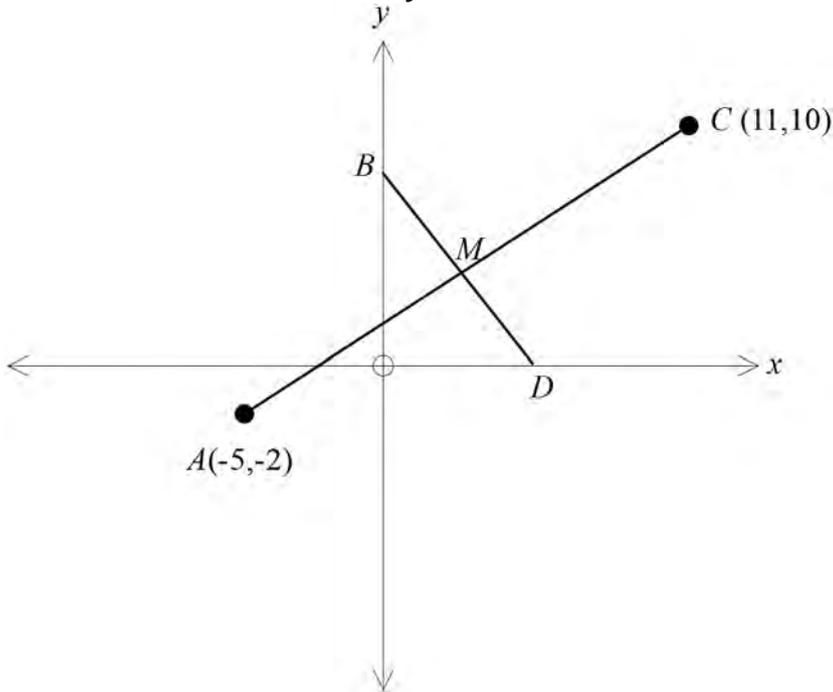
**Question 12** (15 marks) Start a new booklet.

a) Given that  $\int_0^4 kx + 2 dx = 12$ , and  $k$  is a constant, find the value of  $k$ .

2

b)  $A(-5, -2)$  and  $C(11, 10)$  are two points on the number plane.

$M$  is the midpoint of  $AC$  and the perpendicular bisector of  $AC$  meets the  $x$  axis at  $D$  and the  $y$  axis at  $B$ .



- i. Find the coordinates of  $M$ . 1
- ii. Show that the equation of the perpendicular bisector of  $AC$ ,  
i.e. line  $BMD$ , is  $4x + 3y - 24 = 0$  2
- iii. Hence find the coordinates of the points  $B$  and  $D$ . 2
- iv. Show that the quadrilateral  $ABCD$  is a rhombus. 2

c) Chairs are arranged in rows in front of a stage in a concert hall, so the row closest to the stage is the first row. Each row has two more chairs than the row in front of it. There are forty-two chairs in the tenth row.

- i. How many chairs are in the first row? 2
- ii. The seating arrangement has a total of 680 chairs.  
How many rows of chairs are in the concert hall? 3
- iii. How many chairs are in the last row? 1

**Question 13** (15 marks) Start a new booklet.

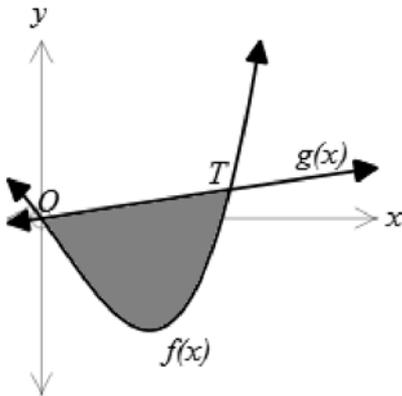
- a) The population  $P(t)$  of turtles in a conservation park is given by:

$$P(t) = 200 - 75 \sin\left(\frac{\pi t}{3}\right).$$

where  $t$  is time in months.

- i. Find all times during the first 12 months when the population equals 275 turtles. 2
- ii. Sketch the graph of  $P(t)$  for  $0 \leq t \leq 12$ . 2

- b) The diagram shows the graphs of the function  $g(x) = 3x$  and  $f(x) = 5x^3 - 5x^2 - 27x$ . The graphs meet at  $O$  and  $T$ .



- i. Find the  $x$ -coordinate of  $T$ . 1
  - ii. Find the area of the shaded region between the graphs of the functions. 3
- c) Tina borrows \$5000 at 1.5% per month reducible interest and pays the loan off in equal monthly instalments. Tina is to repay the loan in 3 years. 3  
Calculate the value of each monthly instalment.



**Question 14** (15 marks) Start a new booklet.

- a) A particle travels so that its displacement (
- $x$
- metres), after
- $t$
- seconds is given by:

$$x = 12t - 3t^2 .$$

- i. Where is the particle 3 seconds after it starts? **1**
- ii. When does the particle turn around? **1**
- iii. How far does the particle travel during the first 5 seconds? **2**
- iv. Find the greatest speed during the first 5 seconds. **1**

- b) A cylinder is to be cut from a solid sphere.

The diagram below shows a cross section of the sphere and cylinder.

The sphere has a diameter of 8 cm. The cylinder has a height of  $h$  cm and a radius of  $r$  cm.

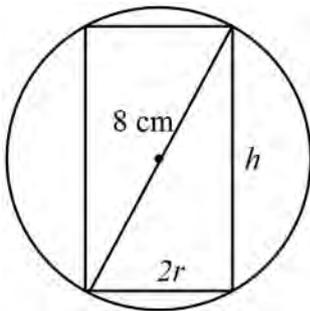


Diagram is NOT drawn to scale.

- i. Show that the volume ( $V$ ) of the cylinder is given by: **2**
- $$V = \pi \left( \frac{64 - h^2}{4} \right) h$$
- ii. Find the value of  $h$  such that the volume of the cylinder is a maximum. **3**

- c) On an island, the population
- $P$
- after
- $t$
- years is given by:
- $P = P_0 e^{kt}$
- .

The initial population of the island is halved in 25 years.

- i. Show that  $k = \frac{\ln 0.5}{25}$  **1**
- ii. How long will it take for the population to reduce from 5000 people to 2000 people? **2**
- iii. What percentage of the original population will be present after 75 years? **2**

**Question 15** (15 marks) Start a new booklet.

- a) i. Copy this table and complete it, leave your answers as fractions. 1

$x$	1	2	3	4	5
$\frac{2}{x(x+1)}$					

- ii. Use the 5 functional values from part i, and Simpson's rule, to find an approximation to 2

$\int_1^5 \frac{2}{x(x+1)} dx$ . Write your approximation using two decimal places.

- iii. Show that  $\frac{2}{x} - \frac{2}{x+1} = \frac{2}{x(x+1)}$ . 2

- iv. Deduce the value of the integral in part ii, correct to two decimal places. 2

- b) In the diagram,  $ABCD$  is a quadrilateral and  $BD$  is a diagonal.  
 $CB = 8$  cm,  $AB = 9$  cm,  $AD = 6$  cm and  $BD = 12$  cm.  $\angle DAB = \angle CBD$ .

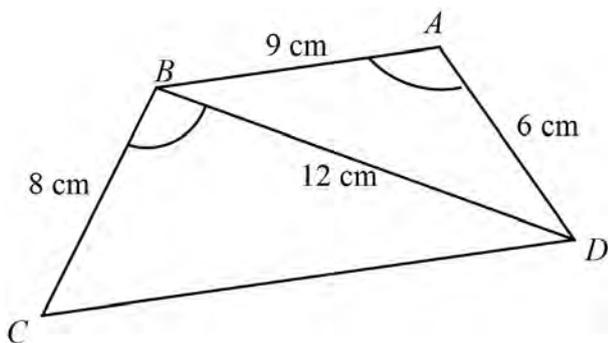
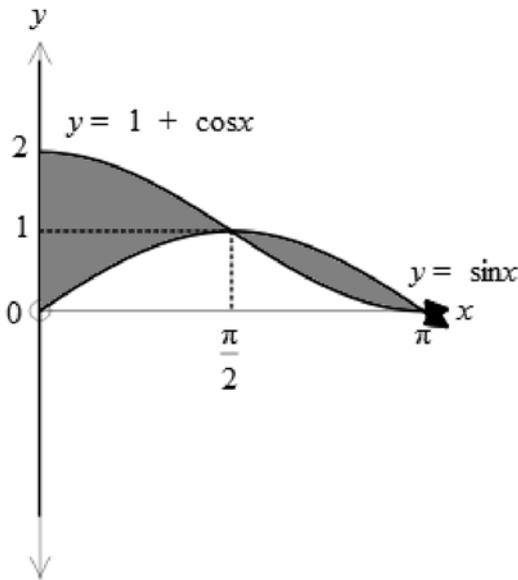


Diagram is NOT drawn to scale.

- i. Prove triangle  $ABD$  and  $BDC$  are similar. 2
- ii. Find the length of  $CD$ . 2
- iii. Prove that  $AB$  and  $CD$  are parallel 1

- c) The graphs of  $y = \sin x$  and  $y = 1 + \cos x$  are shown intersecting at  $x = \frac{\pi}{2}$  and  $x = \pi$  3

Calculate the total area of the two shaded regions.



**Question 16** (15 marks) Start a new booklet.

- a) The region bounded by the curve  $y = \sec x$ , the lines  $x = \frac{\pi}{4}$  and  $x = \frac{\pi}{3}$  is rotated through one complete revolution about the  $x$  axis. 3

Find the volume of the solid of revolution. Give your answer in exact form.

- b) The acceleration of a particle is given by:

$$\ddot{x} = -12e^{-2t}$$

where  $x$  is displacement in metres and  $t$  is time in seconds.

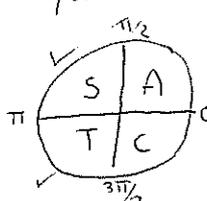
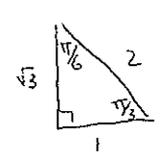
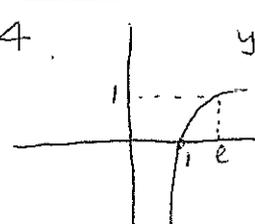
Initially its velocity is  $7 \text{ ms}^{-1}$  and its displacement is 4 m.

- i. Show that the velocity of the particle is given by:  $\dot{x} = 6e^{-2t} + 1$  2
- ii. Graph the velocity with respect to time. 2
- iii. Find the displacement when  $t = 3$  seconds. 2

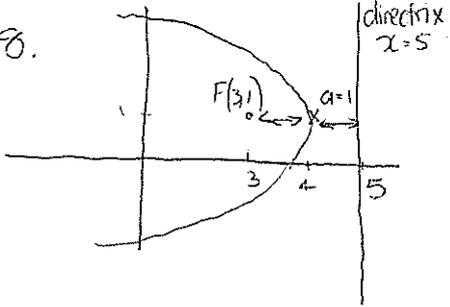
- c) Consider the function  $y = 1 + 3x - x^3$ , for  $-2 \leq x \leq 3$ .
- i. Find all stationary points and determine their nature. **3**
  - ii. Find the point of inflexion. **1**
  - iii. Sketch the curve for  $-2 \leq x \leq 3$ . Do not find the  $x$ - intercepts. **1**
  - iv. What is the minimum value for the curve over the stated domain? **1**

END OF THE EXAMINATION

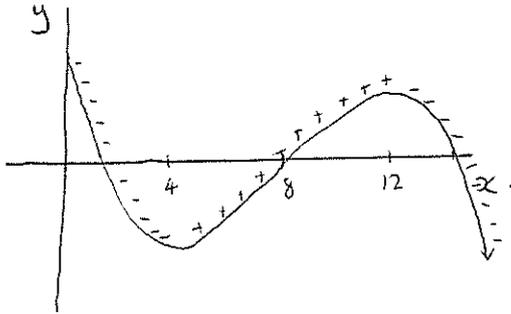
Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
<p><u>Multiple Choice</u></p> <p>Q1 Factorise, then simplify</p> $\frac{(x+6)(\cancel{x-6})}{(\cancel{x-6})} = x+6 \quad \text{(B)}$	
<p>Q2. Quadratic Formula.</p> <p><math>a=3 \quad b=-7 \quad c=-1</math></p> $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-1)}}{2(3)}$ $x = \frac{7 \pm \sqrt{49+12}}{6} = \frac{7 \pm \sqrt{61}}{6} \quad \text{(D)}$	
<p>Q3. Solve, using the 4 quadrants.</p> $2 \cos x = -\sqrt{3}$ $\cos x = -\frac{\sqrt{3}}{2}$ <p><math>x = \pi - \pi/6</math> and <math>\pi + \pi/6</math></p> <p><math>x = \frac{5\pi}{6}</math> and <math>\frac{7\pi}{6}</math></p>  <p>2nd + 3rd Quadrants</p>  <p>(B)</p>	
<p>Q4.</p> <p><math>y = f(x) = \log_e x</math></p>  <p>domain: <math>x &gt; 0</math></p> <p>range: all real y</p> <p>(D)</p>	

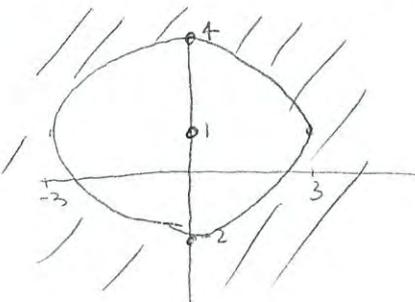
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Suggested solution(s)	comments
<p>Q5. Function of a Function - Chain Rule</p> $y = (e^{3x} + 1)^{-2}$ $\frac{dy}{dx} = -2(e^{3x} + 1)^{-3} \times 3e^{3x}$ $\frac{dy}{dx} = -6e^{3x}(e^{3x} + 1)^{-3} \quad \text{(C)}$	
<p>Q6. <math>x, y</math> <math>a=3</math> <math>b=-2</math> <math>c=10</math>  <math>(4, 5)</math> <math>3x - 2y + 10 = 0</math></p> $d = \frac{ ax + by + c }{\sqrt{a^2 + b^2}} = \frac{ 3(4) - 2(5) + 10 }{\sqrt{(3)^2 + (-2)^2}}$ $d = \frac{12}{\sqrt{13}} \quad \text{(A)}$	
<p>Q7. change index form to log form</p> $\log_5 20 = x$ $\log_5 (5 \times 4) = x$ $\log_5 5 + \log_5 4 = x$ $1 + \log_5 4 \quad \text{(D)}$	
<p>Q8.</p>  <p>vertex <math>(4, 1)</math></p> $(y-h)^2 = -4a(x-k)$ $(y-1)^2 = -4(x-4) \quad \text{(A)}$	

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
<p>Q9. </p> <p>gradient indicated by + &amp; - signs.</p> <p>"approximate" due to scale.</p> <p>function is increasing between <math>x=8</math> and <math>x=12</math></p> <p>rate is increasing between <math>x=4</math> and <math>x=8</math></p> <p>rate is decreasing between <math>x=8</math> and <math>x=12</math></p> <p style="text-align: right;">(C)</p>	
<p>Q10. <math display="block">\int_3^5 4x^3 dx = \left[ \frac{4x^4}{4} \right]_3^5</math></p> $5^4 - 3^3 = 544$ <p style="text-align: right;">(C)</p>	
<p>Q11. a) 0.536 (3.5.F)</p> <p>b) <math display="block">\lim_{x \rightarrow 3} \frac{x^2(x-3)}{(x-3)} = \lim_{x \rightarrow 3} x^2 = 9</math> (1) (1)</p> <p>c) <math display="block">y = (1 + \tan x)^4</math></p> $\frac{dy}{dx} = 4(1 + \tan x)^3 \times \sec^2 x$ (1) $\frac{dy}{dx} = 4 \sec^2 x (1 + \tan x)^3$ (1)	

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
<p>Q11d) <math>y = x \ln x</math></p> $\frac{dy}{dx} = (1) \ln x + x \left(\frac{1}{x}\right) \quad (1)$ $\frac{dy}{dx} = \ln x + 1 \quad (1)$ <p>e) <math>\int 4x e^{x^2+1} dx = 2e^{x^2+1} + C \quad (1)</math></p> <p>f) <math>\int_0^2 \frac{3x}{x^2+1} dx = \left[ \frac{3}{2} \ln(x^2+1) \right]_0^2 \quad (1)</math>  <i>log function</i></p> $\frac{3}{2} \ln 5 - \frac{3}{2} \ln 1 = \frac{3}{2} \ln 5 \quad (1)$ <p><i>correct substitution</i></p> <p>g) circle centre <math>(0,1)</math> radius 3 <math>(1)</math></p>  <p>test <math>(0,0)</math></p> $0^2 + 1^2 \leq 9$ <p><math>\therefore</math> outside of circle required <math>(1)</math></p>	<p>Do not write <math>\frac{4x e^{(x^2+1)}}{2x}</math></p> <p>You will be penalised in HSC.</p> <p>Students did not check answers by differentiating after finding integral.</p>
<p>Q12 a) <math>\left[ \frac{kx^2}{2} + 2x \right]_0^4 = 12 \quad (1)</math></p> $(8k+8) - (0+0) = 12$ $8k = 4$ $k = \frac{1}{2} \quad (1)$	<p>Need to distinguish constants from <math>x</math> variables.</p> <p>Constants are not changed by differentiation or integration</p>

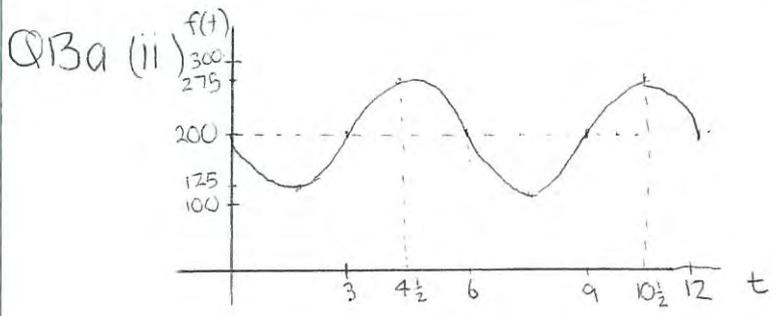
Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
<p>Q12 b) (i) Midpoint of AC = <math>\left(\frac{-5+11}{2}, \frac{-2+10}{2}\right)</math></p> <p style="text-align: center;"><math>M = (3, 4)</math> ①</p> <p>(ii) gradient of AC = <math>\frac{10--2}{11--5} = \frac{12}{16} = \frac{3}{4}</math></p> <p>gradient of BD = <math>-\frac{4}{3}</math> (<math>m_1 m_2 = -1</math>)</p> <p>using (3, 4) and <math>m = -\frac{4}{3}</math> ①</p> <p><math>y - 4 = -\frac{4}{3}(x - 3)</math></p> <p><math>3y - 12 = -4x + 12</math> ①</p> <p><math>4x + 3y - 24 = 0</math></p> <p>(iii) cuts x axis when <math>y = 0</math></p> <p><math>4x - 24 = 0</math></p> <p><math>x = 6</math> D(6, 0) ①</p> <p>cuts y axis when <math>x = 0</math></p> <p><math>3y - 24 = 0</math></p> <p><math>y = 8</math> B(0, 8) ①</p> <p>(iv) midpoint of BD = <math>\left(\frac{6+0}{2}, \frac{0+8}{2}\right)</math></p> <p style="text-align: center;"><math>(3, 4)</math> ①</p> <p>Since diagonals bisect each other at <math>90^\circ</math>, ABCD is a rhombus ①</p>	<p>Some students calculated the sides to be equal.</p>

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
<p>Q12 c) <math>a, a+2, a+4, \dots</math> A.P</p> <p>(i) <math>T_n = a + (n-1)d</math>  <math>42 = a + 9(2)</math> ①  <math>a = 42 - 18 = 24</math> chairs ①</p> <p>(ii) <math>S_n = 680</math>  <math>S_n = \frac{n}{2} (2a + (n-1)d)</math>  <math>680 = \frac{n}{2} (2(24) + (n-1)2)</math> ①  <math>680 = n(24 + n - 1)</math>  <math>680 = n(n + 23)</math>  <math>n^2 + 23n - 680 = 0</math> ①  <math>(n + 40)(n - 17) = 0</math>  <math>n \neq -40</math> <math>n = 17</math> rows. ①</p> <p>(iii) <math>T_{17} = 24 + 16(2)</math>  <math>T_{17} = 24 + 32 = 56</math> chairs ①</p>	<p>Well done.</p> <p>Some students did not get whole number answers. This indicates an error.</p> <p>LO</p>
<p>Q13 a) <math>P(t) = 275</math>  <math>200 - 75 \sin\left(\frac{\pi}{3}t\right) = 275</math>  <math>\sin\left(\frac{\pi}{3}t\right) = -1</math> ①  <math>\frac{\pi}{3}t = \frac{3\pi}{2}, \frac{7\pi}{2}, \dots</math>  <math>t = 9/2, 21/2</math> ①  <math>4.5</math> and <math>10.5</math> months.</p>	<p>Many students could not solve this.</p>

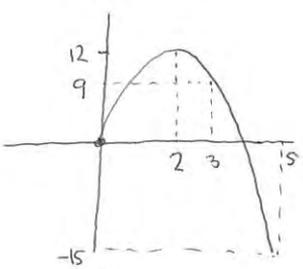
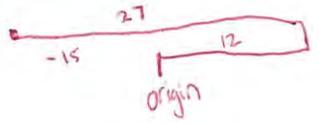
Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
<p>Q13a (ii)</p>  <p style="text-align: right;">②</p> <p>b) (i) <math>g(x) = 3x</math>  <math>f(x) = 5x^3 - 5x^2 - 27x</math></p> <p><math>g(x) = f(x)</math> at point of intersection T</p> $5x^3 - 5x^2 - 27x = 3x$ $5x^3 - 5x^2 - 30x = 0$ $5x(x^2 - x - 6) = 0$ $5x(x - 3)(x + 2) = 0$ <p><math>x = 3</math>    <math>x \neq -2</math> in domain shown.</p> <p>When <math>x = 3</math>    <math>y = 9</math>    <math>(3, 9)</math>    ①</p> <p>(ii) <math>g(x)</math> is above <math>f(x)</math></p> $\int_0^3 3x - 5x^3 + 5x^2 + 27x \, dx$ $\int_0^3 -5x^3 + 5x^2 + 30x \, dx$ $\left[ \frac{-5x^4}{4} + \frac{5x^3}{3} + \frac{30x^2}{2} \right]_0^3$	<p>Curve oscillates about 200. Need labels and scales</p> <p>The curve underneath is subtracted in the integral.</p>
$\left( \frac{-5(3)^4}{4} + \frac{5(3)^3}{3} + \frac{30(3)^2}{2} \right) - 0 = 78.75 \text{ squared units}$ <p style="text-align: right;">①</p>	

Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
<p>Q13 c) 1st month = <math>5000(1.015) - R</math></p> <p>2nd month = <math>5000(1.015)^2 - R(1+1.015)</math></p> <p>3rd month = <math>5000(1.015)^3 - R(1+1.015+1.015^2)</math></p> <p>⋮</p> <p>36th month = <math>5000(1.015)^{36} - R(1+1.015+1.015^2+\dots+1.015^{35})</math></p> <p>After 36th month loan has been repaid</p> <p>∴ <math>R = \frac{5000(1.015)^{36}}{(1+1.015+1.015^2+\dots+1.015^{35})}</math></p> <p><math>R = \\$180.76</math> (1 mark)</p> <p>d) (i) shaded region APQC</p> <p>= Sector OPA - sector OQC</p> <p>= <math>\frac{1}{2}(3)^2\theta - \frac{1}{2}(2)^2\theta</math></p> <p>= <math>\frac{1}{2}5\theta = \frac{5}{2}\theta</math> (1)</p> <p>(ii) <math>\frac{5}{2}\theta = \frac{5\pi}{6}</math></p> <p><math>\theta = \frac{5\pi}{6} \div \frac{5}{2} = \frac{\pi}{3}</math> (1)</p> <p>(iii) <math>\angle DOQ = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}</math> (1)</p> <p>Sector DOQ = <math>\frac{1}{2} \times 2^2 \times \frac{\pi}{6} = \frac{\pi}{3} \text{ cm}^2</math> (1)</p>	<p>See alternative method in Cambridge text</p> <p>Missing steps</p> <p>GP <math>\leftarrow \frac{1.015^{36} - 1}{1.015 - 1} = \frac{1.015^{36} - 1}{.015}</math></p> <p>(1 mark)</p> <p>= <math>\frac{5000(1.015)^{36} \times .015}{(1.015^{36} - 1)}</math></p>

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<u>Suggested solution(s)</u>	<u>comments</u>
<p>Q14 a) (i) <math>x = 12(3) - 3(3)^2</math>  <math>x = 9</math> metres from the origin ①</p> <p>(ii) <math>\ddot{x} = 0</math> whilst turning  <math>\dot{x} = 12 - 6t</math>  <math>12 - 6t = 0</math> when <math>t = 2</math> seconds ①</p> <p>(iii) when <math>t = 0</math> <math>x = 0</math> ①  when <math>t = 2</math> <math>x = 12(2) - 3(2)^2 = 12</math> m  when <math>t = 5</math> <math>x = 12(5) - 3(5)^2 = -15</math> m</p>	<p>• very few people graphed the displacement</p>
<p>total distance = <math>12 + 27 = 39</math> m ①</p>  	<p>• people who did graph managed more correct responses</p>
<p>(iv) greatest speed occurs when the gradient is steepest.  test <math>\dot{x}</math> when <math>t = 0</math> <math>\dot{x} = 6 \text{ ms}^{-1}</math>  test <math>\dot{x}</math> when <math>t = 5</math> <math>\dot{x} = 12 - 6(5) = -18 \text{ ms}^{-1}</math>  greatest speed occurs when <math>t = 5</math> ①</p>	

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<u>Suggested solution(s)</u>	<u>comments</u>
<p>Q14 b. (i) Using Pythagoras' theorem and the diagram provided</p> $(2r)^2 + h^2 = 8^2$ $(2r)^2 = 8^2 - h^2$ $4r^2 = 64 - h^2$ $r^2 = \frac{64 - h^2}{4} \quad \textcircled{1}$ <p>Given Volume of cylinder</p> $V = \pi r^2 h$ $V = \pi \left( \frac{64 - h^2}{4} \right) h \quad \textcircled{1}$ $V = \frac{16\pi h}{4} - \frac{\pi}{4} h^3$ <p>(ii) max occurs when <math>V' = 0</math> and <math>V'' &lt; 0</math></p> $V' = 16\pi - \frac{3\pi}{4} h^2 \quad \textcircled{1}$ $V' = 0 \text{ when } h = \sqrt{16\pi \div \frac{3\pi}{4}}$ $h = \sqrt{\frac{64}{3}} = \frac{8}{\sqrt{3}}$ $\approx 4.6 \text{ cm}$ $V'' = -\frac{6\pi}{4} h < 0 \therefore \text{maximum} \quad \textcircled{1}$ <p>Volume occurs when <math>h = \frac{8}{\sqrt{3}} \text{ cm}</math> <math>\textcircled{1}</math></p>	<p>Many students missed the Pythagoras Application.</p> <p>Some used incorrect formula here</p> <p>test for maximum volume is required. either <math>V'' &lt; 0</math> or <math>\begin{array}{c c c c} h &amp; 4 &amp; 46 &amp; 5 \\ \hline V' &amp; + &amp; 0 &amp; - \end{array}</math></p> <p>4.62cm also accepted.</p>

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<u>Suggested solution(s)</u>	<u>comments</u>
<p>⑩ 14 c (i) <math>P_{25} = \frac{1}{2} P_0</math></p> $\frac{1}{2} P_0 = P_0 e^{k25}$ $\frac{1}{2} = e^{k25}$ $\ln \frac{1}{2} = k25$ $k = \frac{\ln(0.5)}{25} \quad \text{①}$ <p>(ii) <math>2000 = 5000 e^{\frac{\ln 0.5}{25} t}</math></p> $0.4 = e^{\frac{\ln 0.5 t}{25}} \quad \text{①}$ $\ln(0.4) = \frac{\ln(0.5) t}{25}$ $t = \ln(0.4) \div \frac{\ln(0.5)}{25}$ $t = 33 \text{ years} \quad \text{①}$ <p>(iii) <math>P = 5000 e^{\frac{\ln 0.5}{25} \times 75}</math></p> $P = 5000 e^{3 \ln 0.5}$ $P = 625 \quad \text{①}$ $\% \text{ of original} = \frac{625}{5000} \times 100$ $\% \text{ of original} = 12\frac{1}{2}\% \quad \text{①}$	<p>Rounding wasn't penalised.</p>

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Q15 a(i) $x \mid 1 \mid 2 \mid 3 \mid 4 \mid 5$ $\frac{2}{x(x+1)} \mid 1 \mid \frac{1}{3} \mid \frac{1}{6} \mid \frac{1}{10} \mid \frac{1}{15}$ (1)	
(ii) $A \doteq \frac{1}{3} \left( 1 + \frac{4}{3} + \frac{1}{6} \right) + \frac{1}{3} \left( \frac{1}{6} + \frac{4}{10} + \frac{1}{15} \right)$ (1) $A \doteq \frac{47}{45} = 1.04$ (2dp) (1)	using a formula incorrectly for multiple applications.
(iii) LHS = $\frac{2}{x} - \frac{2}{x+1}$ $= \frac{2(x+1) - 2x}{x(x+1)}$ (1) $= \frac{2}{x(x+1)}$ $= \text{RHS}$ (1)	
(iv) $\int_1^5 \frac{2}{x(x+1)} dx = \int_1^5 \frac{2}{x} - \frac{2}{x+1} dx$ $= \left[ 2 \ln x - 2 \ln(x+1) \right]_1^5$ (1) $= (2 \ln 5 - 2 \ln 6) - (2 \ln 1 - 2 \ln 2)$ $= 2 \ln 5 - 2 \ln 6 + 2 \ln 2$ $= 1.02$ (2dp) (1)	

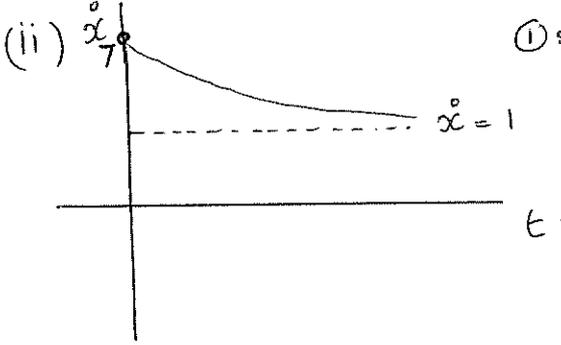
Suggested Solutions, Marking Scheme and Markers' comments

Suggested solution(s)	comments
<p>Q15b)</p> <p>(i) In <math>\triangle ABD</math> and <math>\triangle BDC</math></p> <p><math>\angle DAB = \angle CBD</math> (given) ①</p> <p><math>AB : BD = 9 : 12 = 3 : 4</math></p> <p><math>AD : BC = 6 : 8 = 3 : 4</math></p> <p><math>\therefore \triangle ABD \sim \triangle BDC</math></p> <p>Two sides in same proportion and included angles are equal ①</p> <p>(ii) since <math>\triangle ABD \sim \triangle BDC</math></p> <p>Then <math>DB : CD = 3 : 4</math></p> <p><math>DB = 12</math> <math>12 : CD = 3 : 4</math></p> <p><math>\therefore CD = \frac{12 \times 4}{3} = 16 \text{ cm}</math> ①</p> <p>Corresponding sides on similar triangles have the same ratio ①</p> <p>(iii) since <math>\triangle ABD \sim \triangle BDC</math></p> <p>then <math>\angle ABD = \angle BDC</math></p> <p>alternate angles are equal therefore <math>AB</math> is parallel to <math>CD</math> ①</p>	<p>the abbreviations SAS is only appropriate for congruent triangles.</p>

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Suggested solution(s)	comments
<p>Q15c)</p> $\text{Area} = \int_0^{\pi/2} (1 + \cos x - \sin x) dx +$ $\int_{\pi/2}^{\pi} (\sin x - (1 + \cos x)) dx. \quad (1)$ $\left[ x + \sin x + \cos x \right]_0^{\pi/2} + \left[ -\cos x - x - \sin x \right]_{\pi/2}^{\pi} \quad (1)$ $\left[ \left( \frac{\pi}{2} + \sin \frac{\pi}{2} + \cos \frac{\pi}{2} \right) - (0 + \sin 0 + \cos 0) \right] + \left[ (-\cos \pi - \pi - \sin \pi) - (-\cos \frac{\pi}{2} - \frac{\pi}{2} - \sin \frac{\pi}{2}) \right]$ $\left[ \left( \frac{\pi}{2} + 1 + 0 \right) - (0 + 0 + 1) \right] + \left[ (1 - \pi - 0) - (0 - \frac{\pi}{2} - 1) \right]$ $= \frac{\pi}{2} + 2 - \frac{\pi}{2}$ $= 2 \text{ unit}^2 \quad (1)$	

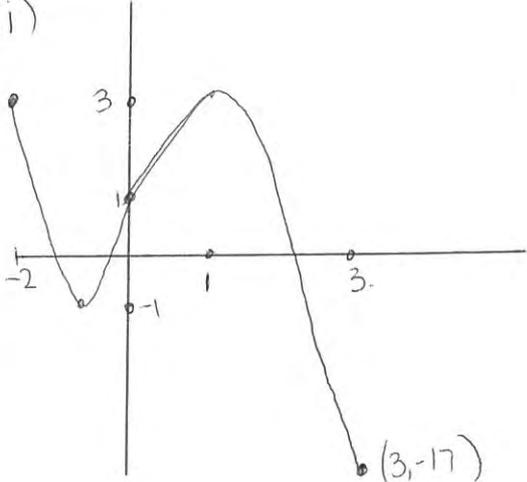
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Suggested solution(s)	comments
<p>① 16 a) <math>V = \pi \int_{\pi/4}^{\pi/3} \sec^2 x \, dx</math> ①</p> <p><math>= \pi \left[ \tan x \right]_{\pi/4}^{\pi/3}</math> ①</p> <p><math>= \pi \left( \tan \pi/3 - \tan \pi/4 \right)</math></p> <p><math>= \pi (\sqrt{3} - 1)</math> cubic units ①</p>	
<p>b) (i) <math>\ddot{x} = -12e^{-2t}</math></p> <p><math>\dot{x} = \int -12e^{-2t} \, dt</math></p> <p><math>\dot{x} = \frac{-12e^{-2t}}{-2} + c</math></p> <p><math>\dot{x} = 6e^{-2t} + c</math> ①</p> <p>When <math>t=0</math> <math>\dot{x} = 7</math></p> <p><math>\dot{x} = 6e^0 + c = 7</math></p> <p><math>c = 1</math></p> <p><math>\dot{x} = 6e^{-2t} + 1</math> ①</p>	
<p>(ii) </p> <p>① shape (0,7)</p> <p>① asymptote.</p>	

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<u>Suggested solution(s)</u>	<u>comments</u>								
<p>Q16 b (iii) <math>x = \int 6e^{-2t} + 1 dt</math></p> $x = \frac{6e^{-2t}}{-2} + t + c$ $x = -3e^{-2t} + t + c$ <p>when <math>t=0</math>, <math>x=4</math></p> $x = -3e^0 + 0 + c = 4$ $c = 7$ $x = -3e^{-2t} + t + 7 \quad \textcircled{1}$ <p>when <math>t=3</math> <math>x = -3e^{-6} + 3 + 7</math></p> $x = 10 - 3e^{-6} \quad \textcircled{1}$									
<p>Q16 c (i) <math>y = 1 + 3x - x^3</math></p> $y' = 3 - 3x^2 \quad \textcircled{1}$ $y'' = -6x \quad \textcircled{1}$ <p>st pnt occur when <math>y' = 0</math> <math>3 - 3x^2 = 0</math></p> $3(1 - x^2) = 0 \quad \text{when } (1, 3) \text{ and } (-1, -1)$ <p>when <math>x = 1</math> <math>y = 3</math> <math>y'' = -6</math> <math>(1, 3)</math> max <math>\textcircled{1}</math></p> <p>when <math>x = -1</math> <math>y = -1</math> <math>y'' = 6</math> <math>(-1, -1)</math> min. <math>\textcircled{1}</math></p> <p>(ii) <math>y'' = 0</math> when <math>x = 0</math></p> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td><math>x</math></td> <td><math>0^-</math></td> <td><math>0</math></td> <td><math>0^+</math></td> </tr> <tr> <td><math>y''</math></td> <td><math>+</math></td> <td><math>0</math></td> <td><math>-</math></td> </tr> </table> <p>change of sign <math>\therefore (0, 1)</math> is a point of inflexion. <math>\textcircled{1}</math></p>	$x$	$0^-$	$0$	$0^+$	$y''$	$+$	$0$	$-$	<p>change of sign test often omitted.</p>
$x$	$0^-$	$0$	$0^+$						
$y''$	$+$	$0$	$-$						

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<p>(iii)</p>  <p>when <math>x = -2, y = 3</math> when <math>x = 3, y = -17</math></p> <p>(iv) Minimum value in the domain <math>-2 \leq x \leq 3</math> is <math>-17</math></p> <p>(1)</p>	<p>one mark was awarded for the shape of curve with tp + st pnt.</p> <p>students over looked the domain <math>-2 \leq x \leq 3</math></p> <p>By over looking the domain the minimum value of <math>-17</math> was often missed.</p>